

The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam

Date: December 12, 2025

Course: ECE 313 Evans

Name: _____
Last, First

- **Exam duration.** The exam is scheduled to last two hours.
- **Materials allowed.** You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks.** Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed.** As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers.** When justifying your answers, reference your source and page number as well as quote the content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab.** No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test.** All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

| Problem | Point Value | Your Score | Topic |
|---------|-------------|------------|---|
| 1 | 18 | | Continuous-Time System Properties |
| 2 | 18 | | Discrete-Time Convolution |
| 3 | 16 | | Continuous-Time Integrators |
| 4 | 14 | | Discrete-Time Filter Design |
| 5 | 16 | | Continuous-Time Sinusoidal Amplitude Modulation |
| 6 | 18 | | Discrete-Time Mystery Systems |
| Total | 100 | | |

Problem 1. Continuous-Time System Properties. *18 points*

Each continuous-time system has input $x(t)$ and output $y(t)$, and $x(t)$ and $y(t)$ might be complex-valued.

Determine if each system is linear or nonlinear, and time-invariant or time-varying.

You must either prove that the system property holds in the case of linearity or time-invariance, or provide a counter-example that the property does not hold. Providing an answer without any justification will earn 0 points.

| <i>Part</i> | <i>System Name</i> | <i>System Formula</i> | <i>Linear?</i> | <i>Time-Invariant?</i> |
|--------------------|--------------------------------------|--|-----------------------|-------------------------------|
| (a) | Finite Impulse Response (FIR) filter | $y(t) = x(t) + x(t - 1)$ for $-\infty < t < \infty$ | | |
| (b) | Inverse | $y(t) = \frac{1}{x(t)}$ for $-\infty < t < \infty$ | | |
| (c) | Differentiator | $y(t) = \frac{d}{dt} x(t)$ for $-\infty < t < \infty$ | | |

(a) Finite impulse response (FIR) filter: $y(t) = x(t) + x(t - 1)$ for $-\infty < t < \infty$. *6 points.*

(b) Inverse: $y(t) = \frac{1}{x(t)}$ for $-\infty < t < \infty$. *6 points.*

(c) Differentiator: $y(t) = \frac{d}{dt} x(t)$ for $-\infty < t < \infty$. *6 points.*

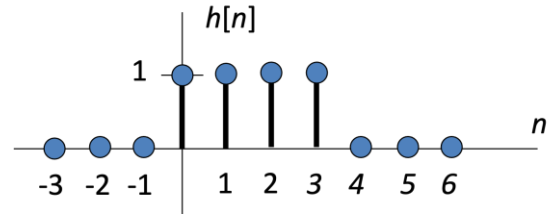
Problem 2. Discrete-Time Convolution. 18 points

Consider a discrete-time linear time-invariant (LTI) system with impulse response plotted on the right of

$$h[n] = \delta[n] + \delta[n - 1] + \delta[n - 2] + \delta[n - 3]$$

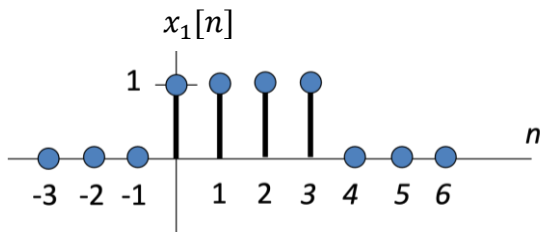
The output of the LTI system is the convolution of the impulse response $h[n]$ and the input signal $x[n]$:

$$y[n] = h[n] * x[n]$$



(a) Let the input signal be $x_1[n] = h[n]$.

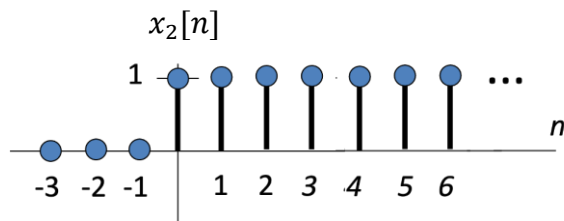
Here, $x_1[n]$ has four non-zero values.



i. Give a formula for $y[n]$. 3 points.

ii. Plot $y[n]$. 3 points.

(b) Let the input signal be $x_2[n] = u[n]$ which is the unit step function.



i. Give a formula for $y[n]$. 3 points.

ii. Plot $y[n]$. 3 points.

(c) Let the input signal be

$$x_3[n] = \cos(\hat{\omega}_0 n) \text{ for } -\infty < n < \infty.$$

Give all possible values for $\hat{\omega}_0$ for

which $y[n] = 0$ for $-\infty < n < \infty$. 6 points.

Problem 3. Continuous-Time Integrator. *16 points*

The integrator is a building block in continuous-time systems.

Denote the input signal as $x(t)$ and the output signal as $y(t)$.

(a) When observing the continuous-time integrator for $-\infty < t < \infty$,

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

The system is linear and time-invariant (LTI).

i. Give a formula for the impulse response. *3 points*.

ii. Give a formula for the step response. *3 points*

iii. Give a formula for the frequency response. *3 points*

iv. Excluding the response to zero frequency, would you describe the frequency response as lowpass, highpass, bandpass, bandstop, or allpass? Why? *3 points*

(b) When observing the continuous-time integrator for $t \geq 0$,

$$y(t) = C_0 + \int_0^t x(\tau) d\tau$$

where C_0 is a real-valued constant.

i. What is the initial condition(s)? *2 points*.

ii. What value(s) should the initial condition(s) have as a necessary condition for LTI properties to hold? *2 points*.

Problem 4. Discrete-Time Filter Design. *14 points.*

This problem asks you to design a discrete-time linear time-invariant (LTI) invariant filter to remove specific frequencies from a signal.

Harmonics can occur due to nonlinear distortion in a system. Design a discrete-time finite impulse response (FIR) filter to remove harmonics of continuous-time frequency, f_0 . Harmonic frequencies of f_0 are $f_0, 2f_0, 3f_0, \dots$ and $-f_0, -2f_0, -3f_0, \dots$

(a) For sampling rate f_s , how many harmonics in positive frequencies, N , would be captured by sampling? *5 points.*

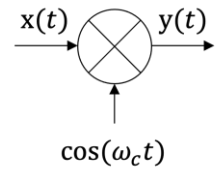
(b) Give formulas for the zeros of the LTI transfer function in the z -domain. *5 points.*

(c) Give the discrete-time input-output relationship for the FIR filter assuming the input signal is $x[n]$ and the output signal is $y[n]$. *4 points*

Problem 5. Continuous-Time Sinusoidal Amplitude Modulation. *16 points.*

Continuous-time sinusoidal amplitude modulation multiplies the input signal $x(t)$ by a sinusoidal signal of fixed frequency ω_c in rad/s to give the output signal $y(t)$ where

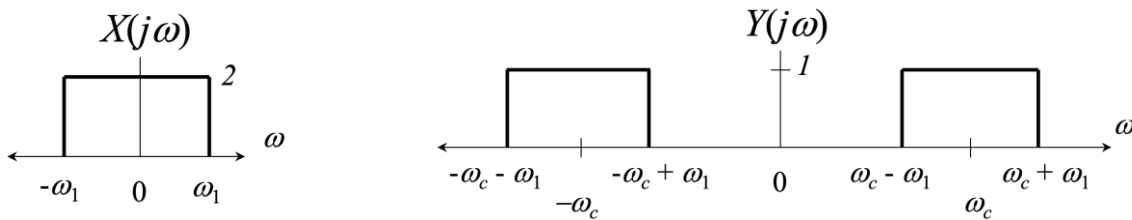
$$y(t) = x(t) \cos(\omega_c t)$$



By taking the Fourier transform of both sides, we obtain the Modulation Property:

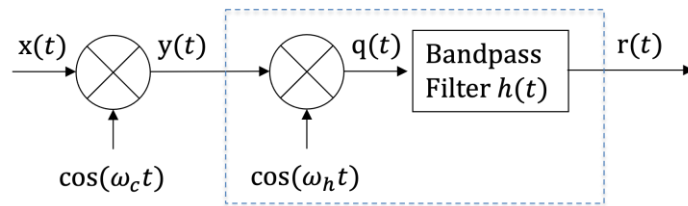
$$Y(j\omega) = \frac{1}{2} X(j(\omega + \omega_c)) + \frac{1}{2} X(j(\omega - \omega_c))$$

The term $\frac{1}{2} X(j(\omega + \omega_c))$ shifts the frequency content of $X(j\omega)$ left in frequency by ω_c and scales the amplitude by $\frac{1}{2}$ and the term $\frac{1}{2} X(j(\omega - \omega_c))$ shifts the frequency content of $X(j\omega)$ right in frequency by ω_c and scales the amplitude by $\frac{1}{2}$. Here's an example using an ideal lowpass spectrum for $X(j\omega)$:

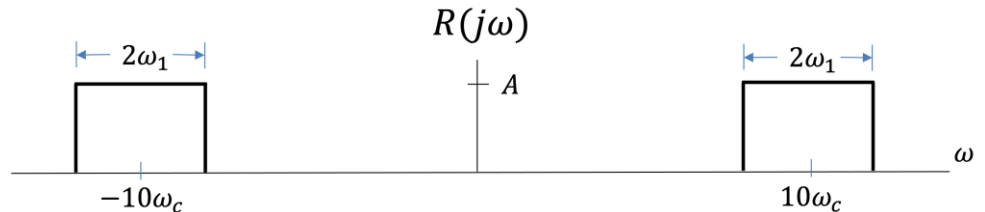


Note $\omega_c > \omega_1$. Please use the above Fourier transforms for $x(t)$ and $y(t)$ throughout this problem.

We can extend this idea to add a second stage of sinusoidal amplitude modulation (in the dashed box) to increase the carrier frequency ω_c to an even higher carrier frequency as shown on the right:

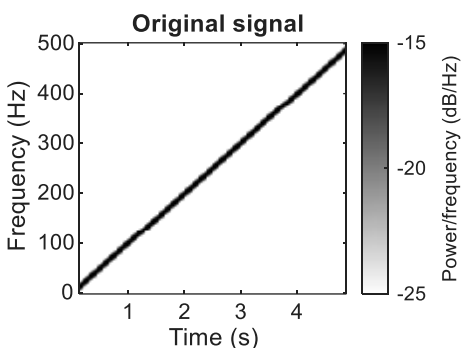


To achieve the frequency content of $r(t)$ shown on the right, complete the following:



- Draw the frequency response of $q(t)$. *4 points*
- What is the value of ω_h in terms of the other parameters in the system? *4 points.*
- Draw the frequency response of the bandpass filter $H(j\omega)$. *4 points.*
- What is the value of A ? *4 points.*

Problem 6. Discrete-Time Mystery Systems. 18 points.



Consider a discrete-time chirp signal $x[n]$ obtained by sampling a chirp signal that sweeps from 0 to 500 Hz.

$$x(t) = \cos(2\pi f_1 t + 2\pi \mu t^2)$$

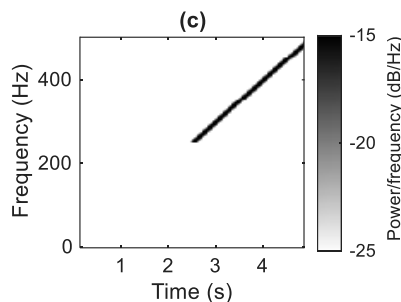
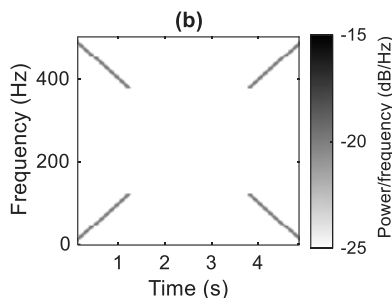
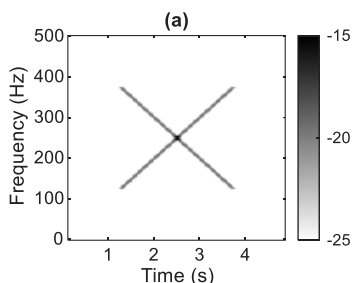
Where $f_1 = 0$, $f_2 = 500$ Hz, $\mu = \frac{f_2 - f_1}{2t_{\max}} = \frac{500 \text{ Hz}}{10\text{s}}$, and $f_s = 1000$ Hz

Assuming $x[n]$ is the input (original signal shown), indicate which of the systems (1-8) would produce each output (a), (b) and (c). Justify your answer.

Here, \downarrow_2 means downsampling by a factor of two and \uparrow_2 means upsampling by a factor of two.

$$\downarrow_2 \{x[n]\} = x[2n] \quad \uparrow_2 \{x[n]\} = \begin{cases} x[m] & m = \frac{n}{2} \\ 0 & n \text{ odd} \end{cases} = \left(\frac{1}{2} + \frac{1}{2} \cos(\pi n) \right) x[m]_{m=\frac{n}{2}}$$

Hint: Draw the spectrograms corresponding to $\downarrow_2 \{x[n]\}$ and $\uparrow_2 \{x[n]\}$.



| | |
|-----|---|
| (1) | $x[n] \rightarrow \uparrow_2 \rightarrow \text{Lowpass} \rightarrow \downarrow_2 \rightarrow y_1[n]$ |
| (2) | $x[n] \rightarrow \uparrow_2 \rightarrow \text{Highpass} \rightarrow \downarrow_2 \rightarrow y_2[n]$ |
| (3) | $x[n] \rightarrow \uparrow_2 \rightarrow \text{Bandpass} \rightarrow \downarrow_2 \rightarrow y_3[n]$ |
| (4) | $x[n] \rightarrow \uparrow_2 \rightarrow \text{Bandstop} \rightarrow \downarrow_2 \rightarrow y_4[n]$ |
| (5) | $x[n] \rightarrow \downarrow_2 \rightarrow \text{Lowpass} \rightarrow \uparrow_2 \rightarrow y_5[n]$ |
| (6) | $x[n] \rightarrow \downarrow_2 \rightarrow \text{Highpass} \rightarrow \uparrow_2 \rightarrow y_6[n]$ |
| (7) | $x[n] \rightarrow \downarrow_2 \rightarrow \text{Bandpass} \rightarrow \uparrow_2 \rightarrow y_7[n]$ |
| (8) | $x[n] \rightarrow \downarrow_2 \rightarrow \text{Bandstop} \rightarrow \uparrow_2 \rightarrow y_8[n]$ |